COMPLEX MATHEMATICS

Jules H. Gilder

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HAYDEN BOOK COMPANY, INC.
Rochelle Park, New Jersey

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USER'S GUIDE FOR THE APPLE II VERSION OF COMPLEX MATHEMATICS

This is part of a series of mathematics programs to help you make better use of the power of the computer.

HARDWARE REQUIREMENTS

Depending on the version of this program that you have purchased, you will need either a Commodore PET with at least 8K of memory, an Apple II with 16K of memory and the Applesoft in ROM card (or 24K if you do not have the ROM card), or a Radio Shack TRS-80 Level II with at least 16K of memory.

LOADING

Use the standard method for loading programs written in BASIC for your machine. The Apple version of this program is written in Applesoft.

USING THE PROGRAM

After the initial sign on, this program is run from an index which will appear on the screen automatically. Instructions for use of each segment of the program will appear on the screen when that segment is called from the index. Note that many of the requests for data entry in the program call for two or more numbers separated by commas to be entered in response to the prompt. For instance:

```
ENTER THE REAL AND IMAGINARY PARTS:

NUMBER 1   ?
```

calls for the real number, followed by a comma, followed by the imaginary number thus:

```
? 2,6.135
```

then RETURN or ENTER depending on your machine.

Complex numbers frequently crop up in engineering calculations, and it is convenient to be able to perform calculations with them on the computer. Unfortunately, this is another area where BASIC is not up to par with FORTRAN. Because of the unique properties of complex numbers, they cannot be handled in the same way as conventional or real numbers are. A complex number is really a pair of numbers "A" and "B" and is expressed as:

A + JB or A + Bl

where J and I have the property that I↑2 or J↑2 is equal to -1. The part of the number that is preceded by the J or followed by the I is called the imaginary part (in the example above B is the imaginary part) while the other part (A) is called the real part.

The series of programs that follow were designed to overcome the lack of complex capability in BASIC and even improve on the ability to perform complex computations that is provided by FORTRAN. This is done by including evaluations that are not offered as standard functions in FORTRAN.

ABSOLUTE VALUE

One capability provided in the FORTRAN complex math package is the absolute value function. This essentially comprises the subroutine that does the polar conversion in the following programs. Unlike the CABS function in FORTRAN, this program will also return the angle as well as the magnitude of the complex number. The magnitude is found by taking the square root of the sum of the squares of the real part and the imaginary part:

Magnitude + (Real↑2 + Imag↑2)↑.5

The angle is determined by calculating the inverse tangent of the imaginary part divided by the real part:

Angle = Arctan (Imag/Real)

This is a tricky calculation to make in BASIC since the result of the inverse tangent function in BASIC is a value in radians in the range of $\frac{\pi}{2}$ to $-\frac{\pi}{2}$. As a result, it is necessary to convert the answer to degrees, and then determine which quadrant the angle is in to get the final result. The conversion to degrees is done in line 640 and the determination of the quadrant and thus the true angle is done in lines 650 to 710.

COMPLEX ADDITION

The addition of complex numbers can be performed by following the general rule:

(A + J B) + (C + J D) = (A + B) + J (B + D)

The program is very general and lets you add any number of complex numbers desired. The addition of the real and imaginary parts is done separately in line 1180 and 1190. Line 1200 determines which sign will be placed in front of the J in the final printout. A minus sign is stored in B$ in line 1145. If the absolute value of the imaginary part is the same as the real value of the imaginary part, then the number must be positive and B$ is reassigned as a plus. Otherwise, the imaginary part is negative and B$ stays as a minus sign. The addition and final printout are all taken care of through line 1230. If you ask for output in polar form, the program will call, as a subroutine, the logic contained in lines 590-760.

COMPLEX SUBTRACTION

The subtraction of complex numbers is very similar to the addition of complex numbers with the only difference being in the signs. It is performed according to the following rule:

(A + J B) — (C + J D) = (A — C) + J (B — D)

Operation of this program is identical to the addition program in that B$ is used to determine the sign of the imaginary component of the answer. The polar format routine is also called in this program.

COMPLEX MULTIPLICATION

Multiplication of complex numbers is somewhat different from the multiplication of ordinary numbers. The real part of the final answer is found by multiplying the imaginary parts of the two numbers and subtracting from the product of the real parts of the two numbers.

The imaginary part of the answer is found by multiplying the real part of the first number by the imaginary part of the second number and adding it to the product of the real part of the second number and the imaginary part of the first number. More clearly stated:

(1) (A + J B)(C + J D) = (AC — BD) + J (BC + AD)

This result can be obtained by treating the two parts of the complex number as two separate numbers. So multiplying C + J D first by A and then by J B gives:

(2) A(C + J D) = AC + J AD

(3) J B(C + JD) = J CB + J↑2 BD

but J↑2 = —1, so adding the two equations together gives the results in Equation 1.

In this program, as in the previous ones, the real and imaginary parts of the final answer are evaluated separately (lines 3120 and 3130) and the ability to get results in polar form is included in the usual subroutines.

COMPLEX DIVISION

Division of complex numbers is performed according to the following rule:

$$(A + JB)/(C + JD) = \frac{(AC + BD) + J\ (BC - AD)}{C{\uparrow}2 + D{\uparrow}2}$$

This is accomplished separately for the real and imaginary parts on lines 4140 and 4150 in the program. The usual polar subroutine is called if you desire it.

Nth ROOTS OF A COMPLEX NUMBER

By substituting the inverse power into De Moivre's equation and expanding it, it becomes possible to calculate the "N," Nth roots of a complex number. The resulting equation that will do this is:

Z↑(1/N) = R↑(1/N)(COS((θ + 2K)/N) + JSIN((θ + 2K)/N)

where K = 0,1,2,3,.........,N—1 and N is the desired root.

In order to implement this equation, it is necessary to call the polar conversion routine twice, once to calculate the results, and a second time to display the results in polar notation.

In order to suppress the printing of the conversion results on the first pass through, variable "SW" has been added. In line 5240 "SW" is tested. If it is equal to 1, that tells the computer that this is the first pass and printing should be skipped. After the results are calculated, SW is set equal to zero and the next time through the subroutine, the results are printed in polar format. For this purpose the polar conversion routine is reproduced at lines 5100 to 5290.

COMPLEX EXPONENTIAL

Calculus often requires the evaluation of the exponential function:

$$EXP\uparrow Z \quad \text{where} \quad Z = A + JB$$

The exponential raised to a complex power can be calculated by using the following formula:

$$EXP\uparrow Z = EXP\uparrow X\ (COS\ Y + J\ SIN\ Y) \quad \text{where} \quad Z = X + J\ Y$$

Evaluation of this formula is done for separate parts in lines 6130 and 6140, and rounding off to 3 decimal places is done in lines 6150 and 6160. Once again polar output capability is provided by the subroutine at line 590.

COMPLEX NUMBER TO A REAL POWER

This is one of the programs that provides more capability than the FORTRAN complex functions do. With it, BASIC can be used to raise a complex number to a real power. The calculation is based on De Moivre's theorem, which states:

$$Z\uparrow N = R\uparrow N\ (COS\ N*\theta + J\ SIN\ N*\theta)$$

where N is the number of roots.

De Moivre's equation is evaluated in lines 7320 to 7340. The same use is made here of the polar conversion routine as was made in Nth Roots of a Complex Number. It is found at line numbers 7100 to 7300.

QUESTIONS AND COMMENTS

We at Hayden are constantly looking for ways to improve our products. We therefore welcome questions and comments, which should be sent to:

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Attention: Software Editor

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